Open Problem Session of STRUCO WORKSHOP 2017

Conditions for a digraph to be partitionable into k cographs

Communiqué par P. Ossona de Mendez.

If G can be partitioned into k cographs, then $\chi(H) \leq k\omega(H)$ for every subgraph H of G.

Problème 1. Find a condition such that every graph satisfying this condition and the condition $\chi(H) \leq k\omega(H)$ for every subgraph H of G, can be partionned into k cographs.

External partition of cubic graphs

Communiqué par J.-S. Sereni.

An *external partition* of a graph G is a bipartition (V_1, V_2) such that $|V_1| = |V_2|$ and $\Delta(G\langle V_i \rangle) \leq 1$ for $i = 1, 2$.

Conjecture 2 (Ban, Linial)**.** Every bridgeless cubic graph has an external partition unless G is the Petersen graph.

An *almost external partition* of a graph G is a bipartition (V_1, V_2) such that $||V_1| - |V_2|| \leq 2$ and $\Delta(G\langle V_i \rangle) \leq 1$ for $i = 1, 2$.

Conjecture 3 (Ban, Linial)**.** Every cubic graph has an almost external partition unless G is the Petersen graph.

Conjecture 4. Let G be a connected cubic graph. If G has no external partition, then $\alpha(G) \geq \frac{n}{2} - 1$.

A first step would be to consider cubic graphs that are the union of two odd cycles joined by a perfect matching.

Homomorphism of cubic graphs into C_5

Communiqué par J. Nesetril.

Conjecture 5. There is an integer k such that every cubic graph G with girth at least k has a homomorphism to C_5.

Note that this is not true for C_{2k+1} for $k \geq 3$. Samal and Devos proved it holds for gap-continuous mapping. (edge version).

The conjecture would imply that the circular chromatic number of cubic graph with large girth is at most $5/2$. But it is not even known whether it is less than 3.

Permutation snarks

Communiqué T. Kaiser

A *permutation graph* is the disjoint union of thwo cycles and a perfect matching between them.

A graph is *cyclically*-5-*edge-connected* if there is no set of 4 edges separates two cycles (i.e. its removal leaves two cyclea in different components).

It was long conjectured that the unique non 3-edge-colourable cyclically-5-edge-connected permutation graph was the Petersen graph. Using computer assistance, other examples have been found but of order 18, 26, 34. This raises the following question.

Problème 6. Is every cyclically-5-edge-connected permutation graph of order $6 \bmod 8$ 3-edge-colourable ?

New proof of a theorem of Volkmann

Communiqué par J.-S. Sereni

Solving a conjecture due to Erdős, Volkmann proved the following.

Theorem 7 (Volkmann)**.** $\chi(G) \leq \max\{|V(H)| - 2\alpha(H) + 2 \mid H \subseteq G\}$.

Volkmann's proof does not really gives the intuition of why this result holds. The idea would be to find an alternative proof.

Generalizing Hall's Theorem

Communiqué par P. Charbit

Let G_1 and G_2 be two bipartite graphs with the same bipartition (A, B). We say that $G_1 \preceq G_2$ if for every $X \subseteq A$ then $|N_{G_1}(A)| \leq |N_{G_2}(A)|$.

Conjecture 8 (Volec)**.** If $G_1 \preceq G_2$, then the number of matchings saturating A in G_1 is no larger than the number of matchings saturating A in G_2.

This conjecture generalizes the celebrated Hall's Theorem.

Finding a subdivision of a fixed digraph

Communiqué par F. Havet

Let F be a fixed digraph. We are interested in the complexity of determining if D contains a (non-necessarily induced) subdivision of F.

A vertex v is *big* if $d^+(v) \geq 3$ or $d^-(v) \geq 3$ or $d^+(v) + d^-(v) \geq 4$. We know that if F is planar without *big* vertices, then the problem is polynomial-time solvable. We know that if all vertices of F are big, then the problem is NP-complete. It is conjectured that if F is non-planar, then the problem is NP-complete.

If F is an orientation of $K_{3,3}$ with a source or a sink, then it is NP-complete. For the other two orientations of $K_{3,3}$, the question is open.

For more results and questions about this topic see [1].

References

[1] F. Havet, A. K. Maia, and B. Mohar. Finding a subdivision of a prescribed digraph of order 4. *Journal of Graph Theory*, to appear. 10.1002/jgt.22174.